1. Consider the following binary operations on $\mathbb{N} \setminus \{0\} = \{1, 2, 3, \ldots \}$

\[ a \oplus b = \max(a, b) \]
\[ a \otimes b = a + b. \]

a. Does there exist $e \in \mathbb{N} \setminus \{0\}$ such that $a \oplus e = a$ for every $a \in \mathbb{N} \setminus \{0\}$? If yes, say what this element is (no need to explain). If not, explain why.

b. Does the equality $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$ hold for any $a, b, c \in \mathbb{N} \setminus \{0\}$? If yes, show that it holds assuming $b > c$. If not, give a counterexample.

c. Which ring axiom does $(\mathbb{N} \setminus \{0\}, \oplus, \otimes)$ fail to satisfy?

2. For the following $R$ and $S$ say if $S$ is a subring of $R$ (without explanation):

a. $R = \mathbb{Z}$ and $S = 4\mathbb{Z}$.

b. $R = \mathbb{Z}$ and $S = \mathbb{Z}_4$.

c. $R = \mathbb{Z}$ and $S$ is the set of even integers.

d. $R = \mathbb{Z}$ and $S = \mathbb{R}$. 