1. 
   a. (10 pt.) Is $f : \mathbb{R} \to \mathbb{R}, f(x) = 2x$ a homomorphism? Explain why.

   b. (20 pt.) Let $F$ be a field. Prove that if $f : F \to F, f(x) = x^2$ is a homomorphism then
      the characteristic of $F$ is 2 (i.e. $1_F + 1_F = 0_F$).
2.

a. (10 pt.) What is the additive order of [27] in \( \mathbb{Z}_{45} \)?

b. (20 pt.) Find the inverse of \( 2x + 3 \) in \( \mathbb{Q}[x]/(x^2 - 2) \).
3.

a. (10 pt.) A 24-hour clock is currently showing 13 : 00. What time will it show in $5^{65}$ hours from now?

b. (20 pt.) Solve the system

\[
\begin{align*}
x & \equiv 17 \pmod{21} \\
x & \equiv 1 \pmod{4}
\end{align*}
\]
4. (10 pt.) Let $F$ be a field. Prove that if $f(x) = x^2 + ax + b \in F[x]$ has no root in $F$ (i.e. there exists no $r \in F$ such that $f(r) = 0$) then it is irreducible. [Hint: Assume $f(x)$ is not irreducible. What does it mean? Why does it imply that $f(x)$ has a root?]